Capital Structure, Hurdle Rates, and Portfolio Choice –
Interactions in an Entrepreneurial Firm

by John Heaton* and Deborah Lucas**

* University of Chicago and NBER
** Northwestern University and NBER

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1. Introduction

In contrast to the shareholders of public corporations, entrepreneurs typically hold large and undiversified equity stakes in their own businesses. In this paper we examine how access to debt financing can influence the choices of entrepreneurs through its affect on the allocation of risk. We focus on the situation of an entrepreneur, who, by including debt in the firm's capital structure, can reduce personal exposure to the firm's idiosyncratic risk. This has implications for optimal capital structure and for the interaction between capital structure and entrepreneurs' portfolio choice, as well as for the hurdle rate used for capital budgeting and hence for the level of investment activity in the economy.

Our analysis builds on the observation that idiosyncratic risk is likely to be priced by undiversified entrepreneurs, but not by the broader market. This wedge between private and market valuation implies that, in the absence of other distortions, entrepreneurs can benefit by issuing risky securities and using the proceeds to reduce their equity stake in the firm. We analyze the case of risky debt, rather than on outside equity or some other risk-sharing contract, because data suggests that debt is the primary source of outside financing for small businesses. Further, theory suggests that debt, because it is less information-sensitive, often dominates equity in settings with asymmetric information. We show in a simple model that in the absence of offsetting factors, the gains from diversification imply maximizing the amount of debt in an entrepreneurial firm's capital structure. When asymmetric information is taken into account, an interior capital structure can be optimal, and there is a tradeoff between improving risk sharing and reducing adverse selection. The ideal amount of debt varies in a natural way with the severity of the information problem and the degree of managerial risk aversion.[1]

In the main analysis we consider a calibrated model in which entrepreneurs are assumed to have limited access to fairly priced debt financing. They can use the proceeds from debt issues

[1] This abstracts from any tax implications of debt, which are outside the scope of this analysis.

both to finance firm projects, and to retain personal wealth outside of their businesses. This creates a direct connection between the optimal capital structure of the firm and the optimal composition of the entrepreneur's portfolio. We characterize the interaction between capital structure and portfolio choice when outside investment opportunities include publicly traded stocks and a risk-free bond. One result is that it can be optimal to hold a leveraged position in one's own business and at the same time hold a large proportion of financial assets in common stocks, even if risk-free investments are available. This is consistent with the observed portfolio allocations of entrepreneurs, who in aggregate hold about 30 percent of the stock market despite their significant exposure to own-firm risk (Heaton and Lucas (2000)).

In this framework, project choice within the firm is an extension of the entrepreneur's optimal portfolio choice problem. The model predicts that the availability of outside debt significantly affects the hurdle rate that entrepreneurs use to evaluate investment projects. Entrepreneurs who have limited access to outside funding may set hurdle rates that are considerably higher than predicted by a traditional asset pricing model like the CAPM. More generally, calibrations show that it is consistent to see entrepreneurs with a modest degree of risk aversion, a high hurdle rate, and sizeable personal investments in the stock market. The model predicts that as risky debt becomes more available the hurdle rate tends to fall. Thus, improvements in access to debt markets can result in an overall increase in welfare when risk aversion inhibits the adoption of worthwhile projects. Increases in wealth relative to firm size similarly reduce hurdle rates.

By linking hurdle rates to idiosyncratic risk, the model provides a potential explanation for several common, if somewhat puzzling, capital budgeting practices. Graham and Harvey (1999) report results on capital budgeting practices from a survey of 392 Chief Financial Officers from a cross-section of firms. They find that small firms and firms with high management ownership are significantly less likely to use the CAPM. Among small firms, CAPM use is inversely related to managerial ownership. In response to a question about whether cash flows and/or discount rates are adjusted for specific risk factors, small firms and firms with high managerial ownership were more likely to indicate that adjustments were made. Poterba and Summers (1995), using a survey of Fortune 500 firms, conclude that even these large firms use hurdle rates that on average exceed what one would expect based on a standard asset pricing model by about 5 percent. Although large firms may rely on inflated hurdle rates in part to mitigate internal control problems, their use may also reflect the undiversified equity positions of many of the senior managers. These managers, in terms of idiosyncratic risk exposure, are in a similar position to entrepreneurs. Finally, the model suggests a potential explanation for the anecdotally high hurdle rates (on the order of 40 percent) required by many venture capitalists. On the other hand, the results appear inconsistent with the findings of Moskovitz and Vissing-Jorgensen (2002) that suggest entrepreneurs accept inexplicably low returns.

The analysis here is related to the literature on capital structure, and also to the literature on entrepreneurship and development. Jensen and Meckling (1976) were among the first offer an explanation for the phenomena of concentrated equity holdings by managers. They examine the optimal scale of investment and capital structure when risk-neutral managers can appropriate rents from outsiders, and find that outside claims would only be issued after managers exhaust their own funds. Leland and Pyle (1977) were the first to emphasize that asymmetric information could cause a risk-averse entrepreneur to hold a large undiversified stake in his own business. In their model, entrepreneurs can signal project quality by their retained ownership stake, enabling them to sell equity to outsiders at a fair price. They assume, however, that debt is risk-free and hence irrelevant for risk sharing. Our model differs in its focus on risky debt rather than outside equity, and because of its emphasis on the interaction with private portfolio holdings. More generally, it has been shown under a variety of assumptions that information problems tend to depress the market price of any new issue of risky securities, including risky debt securities (e.g., DeMarzo and Duffie (1999)). A number of authors (e.g., Hart and Moore (1994), (1995), and

Zweibel (1996)) consider how control issues affect capital structure.[2] While these analyses address important determinants of ownership structure, with the exception of Leland and Pyle (1977), they abstract from managerial risk aversion.

Related models of entrepreneurship include Evans and Jovanovic (1989), Holtz-Eakin, Joulfan, and Rosen (1994), and Gentry and Hubbard (1998), who examine how the decision of whether or not to become an entrepreneur in the first place is influenced by access to capital markets, but under the assumption of risk-neutrality. Two recent papers link managerial risk aversion and agency problems to variation in project characteristics and financing over the business cycle (Rampini (1999), and Levy (2000)). Perhaps most closely related is the work on the influence of improved risk sharing via capital markets on economic growth and income distribution. Among others, Greenwood and Jovanovic (1990), Banerjee and Newman (1991), Obstfeld (1994), and Devereux and Smith (1994) explore these issues. A basic theme that runs through these papers as well as this one is that the ability to trade risk allows specialization in production that can lead to increased investment and growth. A distinguishing feature of the model developed here is the explicit link between debt availability and the choice of project risk and return characteristics.

The remainder of the paper is organized as follows. In Section 2 we present some empirical evidence about small firms' capital structures and the portfolio holdings of entrepreneurs that motivates the focus on external debt as a mechanism for risk sharing, and that is consistent with several of our model's implications. Section 3 presents a simple model that illustrates the tradeoff between risk-sharing and information problems in the context of a market for risky debt. We show the existence, under some parameterizations, of an interior optimal capital structure in the presence of adverse selection. In Section 4, we take the availability of

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[2] More generally, following the work of Modigliani and Miller (1958) many authors have pointed to reasons why capital structure may be relevant (see Harris and Raviv (1991) for a survey).

debt as exogenous, and look at the effect of varying degrees of access to debt markets on hurdle rates and entrepreneurs' portfolio choice in a calibrated model. Section 5 concludes.

2. Evidence on Capital Structure and Portfolio Choice

The analysis below relies critically on the assumption that entrepreneurs are able to reduce personal risk exposure by issuing risky debt, and that debt rather than equity is the marginal source of outside financing. It also has predictions for how entrepreneurs will invest their financial wealth. In this section we present some statistical evidence on these issues, using data from the Fed's 1998 Survey of Small Business Finances (SSBF) and its 1998 Survey of Consumer Finances (SCF).

The SSBF represents the population of small for-profit U.S. businesses with fewer than 500 employees. The survey contains information from 3,561 responding firms, but reported statistics are weighted to represent the overall population of businesses, and to correct for over-sampling of certain types of firms. Since the interaction between business wealth and financial wealth is only relevant for those entrepreneurs with sufficient non-business wealth to also have a portfolio of financial securities, the numbers reported here further condition on net worth (including housing) exceeding the median value of $225,000 unless otherwise indicated. The final sample represents 2.6 million firms. Most non-wealth statistics, however, are similar for the less wealthy firms not included in the reported tabulations. Due to some apparent outliers, the data is Windsorized by dropping the top and bottom 2 percent of certain responses.

Data from the SSBF confirm the high concentration of equity ownership in entrepreneurial firms, and the importance of debt as a source of outside funding. The principle owner holds on average 81 percent of the firm's equity, and the median owner wholly owns the firm. The median ratio of debt to book assets is .61 for firms that have some debt, and .335 overall. The high standard deviation of this ratio (1.9) implies considerable heterogeneity in debt usage. Firms are asked whether a loan had been applied for in the past year. They are also asked whether additional equity financing was obtained, and if that equity came from an owner or outsider. While 22 percent of the firms with some debt indicate that they applied for new debt financing in the previous year, only 6 percent obtained additional equity financing. Furthermore, only 0.5 percent of the additional equity was obtained from outside sources (i.e., 5.5 percent came from existing owners). These findings are consistent with results from earlier Fed surveys of small businesses, as reported in Berger and Udell (1998), Cole and Wolken (1996) and Petersen and Raguhram (1994).

While it seems clear that debt is important for many of the SSBF firms, there is a question of whether and to what extent it transfers risk. As several authors have pointed out (Avery et. al. (1998), Berkowitz and White (2002)) many loans are at least partially secured with a personal guarantee. We find that personal guarantees are important in the SSBF sample, but not universal. Personal guarantees were reported by 61 percent of respondents with loans, with a similar fraction indicating that a loan carried collateral.[3] Furthermore, even if loans are backed by collateral or a personal guarantee, it is likely that some risk remains. The most direct evidence of risk is in the high rates that these firms pay to borrow. The average reported rate paid, not including fees, is 8.9 percent, and the median rate is 8.1%. This represents an approximately 4 percent premium over the one-year Treasury rate, suggesting considerable risk. The relatively high rate of bankruptcy among small businesses also is consistent with a risk transfer to creditors. Finally, debt only shifts risk if it is external. Petersen and Raghuram (2002) find that most debt is from external sources, although 22 to 23 percent comes from owners, family and friends. The inside debt is concentrated among the smallest and youngest firms.

In the model, the predicted hurdle rate is very sensitive to the fraction of an entrepreneur's wealth that must be invested in the firm. The data show the significance of business wealth relative to other wealth for business owners. The statistics on wealth ratios are

[3] Berger and Udell (1995) find that borrowers with longer banking relationships are less likely to pledge collateral, suggesting the importance of relationships in lending.

presented with and without the inclusion of housing equity, since housing wealth may be viewed differently than other financial assets. As is typical of wealth statistics, the large difference between the mean and median reflects the skewness of the wealth distribution. In the calibrations in Section 4, we use the medians as the more representative indicator of typical wealth ratios. Table 2.1 indicates similar total wealth levels, but much higher wealth to business asset ratios for firms without debt. This is consistent with the idea that entrepreneurs with relatively less financial wealth will turn to debt markets to share risk.

Table 2.1: Wealth versus Business Value, SSBF

	Mean	Median
Total Wealth	\$908,416	\$550,000
Total Wealth / Business Value	25.1	4.5
Non-housing Wealth / Business Value	16.0	2.9
Conditional on Using Debt		
Total Wealth	\$962,350	\$620,000
Total Wealth / Business Wealth	10.3	2.4
Non-housing / Business Value	7.5	1.7

The SSBF does not contain detailed information about the composition of financial wealth of business owners. For this information we turn to the SCF, which contains much more complete financial information, although many fewer observations, on business owners. Table 2.2 tabulates the value of stock held by business and non-business owners, for a range of business values and net worth levels. Business owners who do not indicate a controlling interest are classified as non-owners. A striking finding is that although business owners represent a small fraction of the population, they account for approximately 29 percent of total stock holdings.

(This calculation takes the total holdings from Table 2.2 for those with over $10,000 in business value, and divides by total stock market capitalization in 1998.)

Table 2.2: Household Stockholding by Proprietary Business Wealth and Financial Net Worth Conditioning on Active and Controlling Interest (Billions of 1998 Dollars; Number of Households (millions) in parentheses)

Source: 1998 Survey of Consumer Finance.

	Net Worth			
Business Value	<$10K	$10K to $100K	$100K to $1M	>$1M
<$10K	10.1 (33.0)	116.4 (29.2)	1,400.2 (29.0)	2,810.1 (2.3)
$10K-$100K	0.0 (0.2)	3.7 (1.7)	146.2 (2.5)	81.2 (0.2)
$100K - $1M	0.0 (0.0)	0.6 (0.1)	129.1 (2.6)	462.5 (0.6)
>$1M	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	921.9 (0.9)

Table 2.3 shows how stock holdings relative to liquid assets and net worth vary with the ratio of business value to net worth, again conditioning on business owners with more than $225,000 in net worth. Liquid assets include stocks, bonds and cash, while net worth includes liquid asset plus the value of businesses, real estate and pensions, net of personal debt. The tabulations show that the ratio of stocks in liquid assets is fairly invariant to business wealth, but the share of stocks in total wealth sharply declines in business wealth. For the overall population in the SCF with net worth in excess of $225,000, the median ratio of stocks to liquid assets is .63, and the median ratio of stocks to net worth is .22. These statistics are similar to those for business owners in the lowest decile of business value to net worth.

Table 2.3: Stockholdings and Business Holdings

Source: 1998 SCF

	Median Stock / Liquid Assets	Median Stock / Net Worth
Decile Business Value / Net Worth		
1	.87	.24
2	.58	.31
3	.70	.17
4	.57	.18
5	.63	.20
6	.64	.21
7	.65	.12
8	.69	.08
9	.64	.03
10	.65	.02

3. Debt Financing, Risk-Sharing, and Asymmetric Information

In this section we present a simple model in which the optimal debt level balances the benefits of diversification with costs arising from adverse selection. We use a static formulation to simplify the analysis, but expect similar tradeoffs would arise in any dynamic setting. We implicitly rely on these results in Section 4, where it is assumed that only limited debt financing is available, even though more debt would improve risk-sharing and lead to higher value investments.

The analysis focuses on an adverse selection problem rather than moral hazard because the marginal incentive to provide effort is largely unaffected when risk is shared using risky debt. This contrasts with analyses of limitations on outside equity for risk-sharing, which are often motivated by a moral hazard problem, and suggests that one reason for the greater use of debt for risk sharing is its more benign affect on incentives.

As discussed in Section 2, most small or medium-sized firms rely on debt in the form of loans from banks or finance companies, credit cards, trade credit, and loans from family and friends. The analysis applies to any of these financing sources, as long as the lender's recourse to the entrepreneur's personal assets is limited to the assets of the firm.

3.A Optimal Debt with Symmetric Information

Consider an entrepreneur who can finance production in a risky private technology using a combination of endowed wealth and debt.[4] Any wealth not invested in the private production technology can be invested in risk-free debt and publicly traded equities. Consumption is financed from project cash flows and the return on financial investments.

Specifically, the entrepreneur has access to a single factor production technology that is linear in the quantity of capital invested, I, up to some maximum amount of capital that can be productively invested, I_{max}. Total output is ρI, where ρ is a random variable on $[0, \rho_{max}]$. We assume that firm output is distributed independently of stock market returns, so that a well-diversified investor discounts project cash flows at the risk-free rate.[5] For now, the distribution of ρ, $F(\rho)$, is taken to be common knowledge at the time an investment is made. We assume that this private technology cannot be sold directly to outside investors, for instance because its operation requires the special skills of the entrepreneur.

The entrepreneur maximizes the expected utility of consumption:

$$E\{u(c)\} \tag{1}$$

by choosing I, χ, D, S, and B subject to:

$$I = D + \chi$$
$$D \leq D_{max} \leq I$$
$$I \leq I_{max}$$

4 The possibility of outside equity financing is precluded by assumption. Generally, it is more difficult to support equity than debt contracts in environments with adverse selection or moral hazard. The contractual structure is taken is given here, both for simplicity, and because empirically debt is a more important source of outside financing than equity for entrepreneurial firms.

5 A similar analysis would apply if the projects also were subject to systematic risk, but the mathematics would be more complicated.

$$\chi + S + B \leq W$$
$$c = max(\rho I - r(D, \chi)D, 0) + r_s S + r_b B$$

where c is consumption, W is endowed wealth, χ is the amount of personal wealth invested in the firm, S is stock holdings, B is bond holdings, D is the firm's debt, $r(D, \chi)$ is the gross promised return on the debt, r_s is the stochastic gross return on stocks, and r_b is the return on risk-free bonds. The upper bound on debt, D_{max}, allows for the possibility of credit rationing. We further assume that

$$u'(.) > 0, \quad u''(.) < 0, \quad u'(0) = \infty; \quad u(0) = -\infty.$$

Pricing in the debt market is assumed fair due to competition between lenders. Either the debt is repaid in full, or the lender receives the residual value of the project. Since the project has no systematic risk, fair debt market pricing implies that lenders expect to earn the risk-free rate:

$$\int_0^{r(D,\chi)\frac{D}{I}} \rho(D+\chi)dF(\rho) + \int_{r(D,\chi)\frac{D}{I}}^{\infty} r(D,\chi)(D)dF(\rho) = r_b D \tag{2}$$

For exposition, it is convenient to assume that the expected return $E(\rho)$ on the project is sufficiently high, or that endowed wealth W is sufficiently large, so that even in the absence of risk shifting by debt financing the entrepreneur would choose to invest I_{max}. (The benefits of borrowing are even greater when parameters are such that the entrepreneur invests less than the maximum amount without access to debt financing.) The only benefit from borrowing in this case is to reduce exposure to idiosyncratic business risk. With a fixed amount invested, we can re-scale all quantities by dividing by I_{max}. Let $\delta = D/ I_{max}$, so that the choice of δ implicitly determines χ. The asset allocation decision reduces to the choice of what share, s, of remaining wealth $W - I_{max} + D$ is invested in stocks. The promised return on debt depends only on δ instead of on both D and χ, and is denoted by $r(\delta)$. Then the first order conditions for the entrepreneur's maximization problem can be written as:

$$E\{u'(c)(r_s - r_b)\} = 0 \tag{3}$$

$$E\{u'(c)(-r'(\delta)\delta - r(\delta)) \mid \rho - r(\delta) > 0\} + E\{u'(c)(r_b(1-s) + r_s s)\} \geq 0 \tag{4}$$

Equation (3) results from differentiating (1) with respect to *s*, and equation (4) results from differentiating (1) with respect to δ, subject to the budget constraint. The expectation is over both the return on stock and the return on the project. Equation (4) will hold with equality unless the optimal debt ratio is at its upper bound. Substituting equation (3) into (4) yields:

$$E\{u'(c)(-r'(\delta)\delta - r(\delta)) \mid \rho - r(\delta) > 0\} + E\{u'(c)r_b\} \geq 0 \tag{4'}$$

It is convenient to rewrite the fair pricing condition as:

$$\int_0^{r(\delta)\delta} \rho dF(\rho) + \int_{r(\delta)\delta}^{\infty} r(\delta)\delta dF(\rho) = r_b\delta \tag{2'}$$

In the absence of other distortions, these conditions imply that it is optimal for the entrepreneur to maximize the use of fairly priced outside debt, so long as that debt is risky. As shown in Lemma 1 in Appendix A, risk-free debt serves no purpose since it transfers no risk and is fairly priced. The strategy of maximizing leverage minimizes the impact of idiosyncratic risk by effectively allowing the entrepreneur to pool with other entrepreneurs via the financial intermediary or debt market. When output is high revenues are sufficient to repay the debt in full, thereby lowering consumption relative to self-financing. When output is low the firm defaults, but the consumption of the entrepreneur is higher than it would have been in the absence of debt, since more wealth is held in marketable assets outside the firm. The net effect is to transfer consumption from high output to low output states, thereby increasing utility. This argument is formalized in Theorem 1.

<u>Theorem 1</u>: Assume *W* and *F(ρ)* are such that $I = I_{max}$ without borrowing. If debt is fairly priced as in equation (2'), then it is optimal for the entrepreneur to borrow to the maximum extent possible. (Proof in Appendix A.)

In Theorem 1, the scale of investment is assumed to be unaffected by the availability of debt. As one would expect, borrowing further increases welfare when it increases the scale of profitable investment. Due to risk aversion, an entrepreneur may choose to invest less than all his wealth, even when $E(\rho) > r_b$ and the project has only idiosyncratic risk. Lemma 2 in Appendix A provides sufficient conditions for under-investment in the absence access to risky debt.

3.B. Optimal Debt with Adverse Selection

The above analysis suggests that entrepreneurs should borrow as much as possible if they have access to fairly priced debt. In practice small firms often pay a high price for borrowing, and their use of leverage is limited (see Section 2). One likely reason for the limited use of debt is adverse selection. If entrepreneurs have private information about project quality, then high-quality entrepreneurs may avoid or scale back borrowing if it involves pooling with lower-quality entrepreneurs and paying a higher rate.[6] In this section we formalize this intuition, and show that the tension between the desire for diversification and the cost of adverse selection can lead to an interior solution for optimal debt.

To incorporate adverse selection, we assume that each entrepreneur privately observes the distribution of output $F_j(\rho)$ for his firm. Entrepreneurs' investments in marketable assets are also assumed to be private information.[7] Lenders know the aggregate distribution of output $F_A(\rho)$, but not the distribution for individual firms. They also know the distribution of project quality among their borrowers, which in equilibrium may differ from the aggregate distribution of

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[6] Petersen and Rajan (1994) provide evidence that some small businesses to some extent mitigate these problems by developing relationships with bankers.

[7] If portfolio choice were observable, it could reveal the entrepreneur's type. The equilibrium either would unravel, or entrepreneurs would also pool in their portfolio choices. It seems reasonable to assume that portfolios are unobservable, however, since it is relatively easy for an entrepreneur to privately rebalance his personal portfolio after receiving a loan.

quality. Specifically, we assume that there are two quality types in the economy. Output for each type takes on two possible values: 0 with probability $(1\text{-}p_j)$, or ρ with probability p_j , j = high, low, where $p_{high} > p_{low}$. A fraction of entrepreneurs, q, are endowed with the good projects, and *(1-q)* are endowed with the bad projects.

Depending on parameter values, different types of equilibria can be shown to obtain. If the quality differential between the two types is sufficiently small, or if there is a high proportion of good borrowers, then a pooling equilibrium with limited borrowing may exist. For other parameterizations there is a separating equilibrium where, if all projects have a positive NPV (i.e., $p_{low}\rho \geq r_b$), the bad types borrow as much as possible and the good types borrow less at a lower rate. Finally, if adverse selection is sufficiently severe, for instance if a large fraction of firms have negative NPV projects, there may be no equilibrium with positive borrowing. Among these possibilities, the pooling equilibria are interesting because they exhibit the observed phenomenon of limited borrowing at relatively high interest rates.

Theorem 2: For q and p_{low} sufficiently low, and if either $p_{low}\rho < r_b$ or equation (B3) holds, then there exists a pooling equilibrium with all entrepreneurs borrowing $0 \leq \delta^* < 1$. (Proof is in Appendix B).

Adverse selection is just one of several possible explanations for the limited use of debt. Risky debt can also create incentive problems, either in the form of excessive risk taking or reduced effort, or the cost of verifying cash flows can be prohibitive. These problems, however, are less acute for debt than for equity. Any combination of these reasons can be used to motivate the maintained assumption in the next section that the ability to use debt to shift risk is limited.

4. The Effect of Limited Debt on Hurdle Rates and Portfolio Choice

The above analysis shows that the availability of outside debt financing can improve risk-sharing, and thereby increase the propensity of entrepreneurs to invest in risky projects. Here we want to express the change in the propensity to invest in terms of hurdle rates. In practice, whether an investment is worthwhile is often evaluated by checking whether the expected return on the project exceeds a target risk-adjusted hurdle rate. If so, the project is undertaken since it has a positive net present value. The standard textbook advice on how to estimate a hurdle rate is to consider how much systematic risk is embodied in the project. Cash flows are then discounted at a rate adjusted for this risk, for instance, using the CAPM. Standard procedures that ignore idiosyncratic risk presume that well-diversified investors own the firm. Risk-averse investors of a closely held firm, however, can be expected to use a hurdle rate that also reflects the utility cost of idiosyncratic risk.[8] To the extent that the availability of debt financing decreases exposure to idiosyncratic risk, it should lower a firm's effective hurdle rate. In fact for a closely held firm, the appropriate hurdle rate will vary with a number of factors. These include the scale of the project relative to the wealth of the proprietor, the interest rate on debt, the amount of debt available, the risk aversion of the entrepreneur, and the quantity of idiosyncratic and systematic risk embodied in the project.

While it is not surprising that these factors should have an affect, what to our knowledge has not been explored is their quantitative importance. To provide a sense of the magnitudes, we calibrate a simple, partial equilibrium, investment model. The idea behind this analysis is that entrepreneurs are often faced with a menu of potential projects with a variety of risk and expected return characteristics. They are likely to take as given the quantity and pricing of available debt for a particular project or set of projects, and there is often a natural scale to the project (e.g., one new retail store). We want to examine how the minimum acceptable expected rate of return, or

[8] For firms with a combination of well-diversified and poorly diversified investors, there is no obvious characterization of the optimal hurdle rate.

hurdle rate, varies with the assumed risk characteristics of a project, the assumed availability of outside debt, and the risk preferences of the entrepreneur.

Any of the entrepreneur's wealth not used to finance the chosen project can be invested in bonds and stocks. We interpret the risky stock as the market portfolio. Since the hurdle rate is determined by the entrepreneur simultaneously with the share of outside wealth invested in stocks, the assumed return on stocks has an important influence on project choice. We are interested in how the hurdle rate shifts when the entrepreneur can invest in stocks with high return. We also examine the extent to which the entrepreneur wishes to invest in risky stocks even when faced with a highly undiversified position in a project.

4.A. Model Setup

To get quantitative estimates, utility is parameterized using constant relative risk aversion with risk aversion coefficient γ. Entrepreneurs are endowed with wealth W, and a risky investment project. Wealth can be invested in the project, in a risk-free bond, and/or risky stocks. The scale of investment is fixed at I^*; it is a take it or leave it project.[9] Total returns are summarized by $\{(\rho+\Phi)I^*\}$, where ρ is the expected rate of return on the project and Φ is a random variable with mean zero. When a project is not undertaken the entrepreneur invests proportion s_0 of his wealth in stocks. When the project is taken this proportion is s.

The availability of and interest rate on debt are taken as given by the entrepreneur, and are denoted by D and r respectively. The hurdle rate, ρ^*, is the value of ρ such that the entrepreneur is just indifferent to investing in the project and investing exclusively in the financial market. The break-even hurdle rate solves the equation:

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9 It is convenient to fix the scale of investment because if it were assumed a choice variable, the hurdle rate and the scale of the investment would interact.

$$\max_{s_0} E\left\{\frac{[W(r_b + s_o(r_s - r_b))]^{1-\gamma} - 1}{1-\gamma}\right\} - \max_{s} E\left\{\frac{[\max(0,(\rho+\Phi_i)I^* - rD) + (W + D - I^*)(r_d + s(r_s - r_b))]^{1-\gamma} - 1}{1-\gamma}\right\} = 0 \quad .(5)$$

We consider several cases where the investor is excluded from investing in stocks. In those cases we solve (5) for ρ when $s_0 = s = 0$.

Results reported below are under the assumption that the interest rate on debt is fair for a project that has an expected return equal to the hurdle rate, and known risk characteristics. The fair interest rate is determined according to (2), and corresponds to situations in which a separating equilibrium obtains for this project type. Of course in a pooling equilibrium, some entrepreneurs would face a higher-than-fair rate and others a lower-than-fair rate, and this in turn would influence their choice of a hurdle rate

4.B. Calibration

To allow numerical evaluation of (5), several parameters must be calibrated. In setting those parameters we have two objectives in mind. First we wish to examine the implications of the model for the cost of capital of the "typical" entrepreneur. Second, and perhaps more importantly, we are interested in the extent of heterogeneity in stock holding and hurdle rates that is implied by differential exposure to business risk and access to outside sources of funds. For this reason we consider a range of parameterizations that capture the variation observed in the SCF and SSBF.

Preferences:

There is no complete agreement on the degree of risk aversion for the typical entrepreneur. Instead of focusing on single value of this parameter we consider values for γ from 0.5 to 5.

Technology:

There is little available data that can be used to directly calibrate the technological risk faced by an entrepreneur. Instead consider the standard deviation of the return to small portfolios of randomly chosen stocks from CRSP. For example, the standard deviation of the return to a portfolio of 5 stocks is 28% and to a single stock is approximately 50%. The significant idiosyncratic risk faced by an entrepreneur is likely to be within this range. We therefore calibrate the risk faced by the entrepreneur by considering several settings for the standard deviation of project risk. The values we use are 28%, 36% and 50% that we refer to as “low”, “medium” and “high” project risk respectively. We also consider the possibility that the project ends in complete failure by allowing Φ to take on the value of zero with positive probability.

We further assume that Φ takes on three possible values $\{-\rho, \Phi_2, \Phi_3\}$ with probabilities $\{p, (1-p)/2, (1-p)/2\}$. We let p take on the values of 0 or 5%.

Although the volatility of Φ is somewhat arbitrary we can use the implied consumption outcomes of the model to examine the reasonableness of these assumptions. For example Attanasio, Weber and Tanner (2002), and Vissing-Jørgensen (2002) report consumption growth volatility for stock-holders in the range of 10% to 15% per annum. Because we are considering consumption volatility at the initiation of a project these sample statistics likely understate the volatility faced by entrepreneurs. For this reason we will allow consumption volatility to be higher than the reported statistics for stockholders. The consumption data does provide a reasonable starting point, however.

Financial Market Returns

We assume that the fixed bond return is 5%. The stock return is assumed to take on the values -5% and 27% with equal probability. This implies an average return of 11% with a standard deviation of 16%. For comparison we consider examples where the only available financial instrument is the bond with a fixed return of 5%.

Wealth and Debt Levels

The observed ratio of project size to wealth levels varies greatly in the data as we discussed in Section 2. For businesses with some external borrowing the median ratio of wealth to business value is 2.4. The median falls to 1.7 if housing wealth is excluded. Furthermore there is extensive variation in the data around these medians. We capture this heterogeneity by considering values of W/I^* that range from 1 to 3. Debt relative to the assets of the firm, D/I^*, is also assume to vary from 0.3 to 0.7.

4.C. Results

Stock Market Exclusion

Calibration results are reported in Table 4.1 (the tables and figures for this section can be found at the end of the paper), under the assumption that the entrepreneur cannot invest in the stock market. The implied hurdle rate is highly sensitive to the availability of debt. For instance with $\gamma = 2$, the medium risk case, and $p = 5\%$, the hurdle rate varies from 21% when $D/I^* = 0.3$ to 12% when $D/I^* = 0.7$. Notice that even with 70 percent debt, the hurdle rate is significantly higher than the 5% required by diversified investors. At this high required return consumption volatility is not unreasonably high. The hurdle rate is sensitive to the assumed level of risk aversion. Notice, however, that even for very low levels of risk aversion, the hurdle rates are quite high.

Several other properties of the hurdle rate are worth noting. As established by Lemma 1, for parameter values such that debt is risk-free, the hurdle rate is unaffected by the amount of debt since it has no affect on risk sharing. The hurdle rate is sensitive to changes in the assumed project risk, Φ.

The hurdle rate and fair borrowing rate are also quite sensitive to the ratio of wealth to the required investment, W/I^*. Table 4.2 gives an example of what happens to hurdle rates and the fair borrowing rate under the assumption of medium risk, p = 5%, D/I^* = .5, and the wealth to project size ratio increases from 1 to 1.5. When wealth is small relative to required investment, effective risk aversion is high. This results in a high hurdle rate and a relatively high payoff in the bad states, which in turn implies a low interest rate on the debt. As wealth grows, effective risk aversion decreases, resulting in a lower hurdle rate and a higher fair interest rate. Similar effects arise in Table 4.1 when the assumed coefficient of risk aversion is decreased. These results are consistent with the empirical finding of Gentry and Hubbard (1998), who find that wealth is a significant factor in determining which households enter into entrepreneurial activity and which exit from entrepreneurial activity.

The results of Table 4.2 highlight the interaction between private wealth, effective risk aversion, and the fair rate of return on debt. Entrepreneurs with relatively high wealth have a lower hurdle rate due to their lower effective risk aversion. Because of this, they accept relatively risky projects, and their debt tends to be riskier. Thus wealthier entrepreneurs may be more likely to be in the position of receiving risk-sharing benefits from debt, even though the risk-sharing benefits are less important to them. Poorer entrepreneurs demand a higher expected return in compensation for putting proportionally more of their wealth at risk, making the same amount of debt less risky and hence less beneficial in terms of risk-sharing.

The Interaction of Project Choice and Portfolio Choice

We now allow the entrepreneur to invest in the market portfolio along with bonds. We do not place any limitation short sales so that the entrepreneur is allowed to finance stock holdings with a short bond position.

Table 4.3 reports results for the case where wealth is fixed at a level equal to the total investment required for the project, and risk aversion is fixed at 2. The table displays the hurdle rate, fair borrowing rate, portfolio share in stocks if a project with the breakeven hurdle rate is adopted, and portfolio share in stocks with no project. The column labeled "S/W" reports the ratio of stock holding to total wealth. Also reported in the table is the correlation between consumption and stock returns. This is given by $\rho(c,r_s)$.

The debt rates and hurdle rates reported in Table 4.3 are somewhat higher than in Table 4.1 due to the fact that the investor has an additional financial market opportunity. Even with a relatively small probability of a default state, hurdle rates in all cases are significantly greater than the 5% required in a complete market. Hurdle rates range from 8% with low project risk and high availability of debt, to 31% with high project risk and low availability of debt.

Table 4.4 reports hurdle and debt rates when wealth is 1.5 times required investment, and all else is as in Table 4.3. Hurdle rates are still significantly higher than the risk-free rate although as in the comparison between Tables 4.1 and 4.2, increasing the ratio of wealth to investment decreases hurdle rates, but has an ambiguous affect on debt rates.

Changing either the level of debt or the level of wealth changes the business exposure faced by the entrepreneur. As a result these variables are important determinants of the cross-sectional heterogeneity in stock holdings and hurdle rates. To illustrate the potential magnitudes Figures 1and 2 display the response of the hurdle rate and share of stock in liquid wealth, respectively, as a function of wealth and debt. These figures are drawn for the case where γ =2, the high risk case with p = 5% and where the investment is assumed to be 1. Notice that for some values of wealth, a change in the assumed level of debt has a very large effect on hurdle rates and on the share of stock in liquid wealth. This can be explained by the fact that increasing debt reduces losses preferentially in bad states of the world, whereas increasing wealth raises baseline consumption but does not tilt the risk profile as much. These figures also illustrate the substantial heterogeneity predicted in hurdle rates and stock holdings as a function of wealth and debt levels.

Because of the low assumed coefficient of relative risk aversion, the entrepreneur always takes a levered position in stocks for these parameters, even when exposed to the idiosyncratic risk of the project. In fact, the share of stock in the entrepreneur's portfolio is actually larger when a project is undertaken than when it is not.[10] It should be noted, however, that the absolute dollar amount invested in stocks decreases when the project is undertaken, since less money is available for portfolio investment. To see this compare columns 6 and 8 of Table 4.4 with those of Table 4.5. In each case the share of stock in total wealth is lower when the project is undertaken.

As emphasized by Heaton and Lucas (1997) and many others, the presence of human capital that provides a relatively risk-free flow of consumption, for instance in the form of wages, can have a significant effect on predicted asset holdings in models with uninsurable risks. This occurs in our setting as well if we allow there to be a consumption floor that we think of as being generated by the returns to human capital. We set this floor to be 20% of current wealth. For comparison, Heaton and Lucas (2000) report that the median value of capitalized labor income for wealthy households is 26%. Table 4.6 reports the results. Notice that the level of stock holding increases significantly in the presence of floor income.

Overall, the calibrations support the idea that reasonable levels of risk aversion can result in stockholding approaching the levels observed in the data even in a situation where the entrepreneur holds a substantial position in an undiversified business. Furthermore consumption variation is approaching the level observed within wealthy households. At the same time the model produces a very significant hurdle rate for privately held businesses.

10 One might expect that the introduction of a risky project would make the entrepreneur, whose preferences exhibit prudence (Kimball (1990)), effectively more averse to stock market risk. Although this would be true of a zero mean gamble, because the project has a positive expected return there is an offsetting wealth effect that makes the reaction to the increased risk ambiguous.

5. Discussion and Conclusion

In this paper, we have developed a simple model in which to analyze the interactions between capital structure, project choice, and portfolio choice that result from entrepreneurial risk aversion. The main qualitative findings are: (1) the ability to issue risky debt improves risk sharing since it allows entrepreneurs to diversify, (2) an interior capital structure may obtain when entrepreneurs trade off diversification benefits against information problems, (3) hurdle rates decrease with the availability of risky debt, and (4) entrepreneurs choose to hold risky stock in their portfolios despite their undiversified stake in their own firm. The first three findings are intuitively plausible, but difficult to verify empirically. The fourth is consistent with observed practice. Quantitatively, the model suggests that under the assumption of moderate risk aversion and projects with risk on order of that of holding an individual stock, undiversifiable risk has a large and positive influence on hurdle rates. This effect is significantly mitigated, but not eliminated, by the availability of risky debt. It also is reduced, but to a lesser extent, by the presence of private wealth that can be held outside the firm in a diversified portfolio. For apparently reasonable parameter values, the model produces hurdle rates that exceed those predicted by the CAPM by amounts consistent with deviations from the CAPM found in practice by Poterba and Summers (1995).

These results suggest that the standard academic advice on how to set hurdle rates may be inappropriate for many enterprises. Unfortunately once the assumption of perfect diversification is relaxed, the theoretically correct hurdle rate becomes a function of many difficult to determine quantities, including risk aversion parameters, idiosyncratic risk, and debt availability. Despite these difficulties, it may be possible to improve on the current practice of abstracting entirely from idiosyncratic risk effects. One approach might be to use observed deviations of hurdle rates from CAPM predictions to calibrate a more highly parameterized model like the one here, taking into account what is known about an individual entrepreneur's situation. More generally, reexamining the cross-section of data on common capital budgeting practices might reveal regularities in practices that can be attributed to differences in idiosyncratic risk exposure.

The model could be generalized in a number of dimensions. Adding a dynamic element would be interesting in order to explore the potential for dynamic hedging strategies, particularly if project risks and market risks were assumed to have a more complicated correlation structure. A more challenging extension would develop the general equilibrium implications, both for asset prices and for decision rules, of the fact that some market participants price idiosyncratic risk. This is a topic of ongoing research.

Appendix A

Lemma 1: The entrepreneur is indifferent to issuing risk-free debt.

Proof of Lemma 1: For quantities of debt such that the debt is risk-free, (4') holds with equality since $r'(\delta) = 0$ and $r(\delta) = r_b$.

The intuition behind Lemma 1 is that when the debt is risk-free, the entrepreneur is indifferent to it since any increase in debt at the firm level can be offset by a purchase of risk-free bonds in the entrepreneur's portfolio.

Proof of Theorem 1: From equation (2'),

$$r(\delta) = \frac{r_b}{1 - F(r(\delta)\delta)} - \frac{\int_0^{\delta r(\delta)} \rho dF(\rho)}{\delta\left(1 - F(r(\delta)\delta)\right)} \tag{A1}$$

Since ρ is independent of stock returns, the entrepreneur chooses δ and the share of available assets invested in stocks, s, to maximize:

$$\begin{aligned} &\int_0^{\delta r(\delta)} E\left[u(sr_s(W/I_{\max} + \delta - 1) + (1-s)r_b(W/I_{\max} + \delta - 1))\right]dF(\rho) + \\ &\int_{\delta r(\delta)}^{\infty} E\left[u(\rho - r(\delta)\delta + sr_s(W/I_{\max} + \delta - 1) + (1-s)r_b(W/I_{\max} + \delta - 1))\right]dF(\rho) \end{aligned} \tag{A2}$$

The expectation in the integral is over the distribution of stock returns. The first order condition (4') involves the term, dr(δ)/dδ. This can be found by noting that the additional expected revenue that the lender collects must equal the fair return on the incremental debt. Since it is assumed that there is no incremental investment, there is no incremental revenue in any state with partial default. Ignoring terms with the product (dr)(dδ), this implies:

$$(dr)(\delta)\delta(1-F)+r(\delta)(d\delta)(1-F)=r_b(d\delta).$$

Rearranging gives

$$\frac{dr(\delta)}{d\delta}=\frac{r_b-r(\delta)(1-F)}{\delta(1-F)} \tag{A3}$$

Since u'(.) is monotonic, by the intermediate value theorem equation (4') can be rewritten as:

$$\begin{aligned}&\left[F(\delta r(\delta))u'(c_{low})+(1-F(\delta r(\delta))u'(c_{high})\right]r_b \geq \\ &(1-F(\delta r(\delta))u'(c_{high})\left(\frac{r_b}{1-F(\delta r(\delta))}-\frac{\int_0^{\delta r(\delta)}\rho dF(\rho)}{\delta(1-F(\delta r(\delta)))}+\frac{r_b-r(\delta)(1-F(\delta r(\delta)))}{(1-F(\delta r(\delta)))}\right)\end{aligned} \tag{A4}$$

where c_{low} is the consumption in a default state that leads to a product equal to the original conditional integral, and c_{high} is defined similarly for the non-default states. By (A1), the term in parentheses on the right-hand side of equation (A4) reduces to $r_b/(1\text{-}F)$. By the concavity of the utility function $u'(c_{low}) > u'(c_{high})$, so (A4) always holds as an inequality. It follows that δ is set to its upper bound.//

In Theorem 1, the scale of investment is assumed to be unaffected by the availability of debt. As one would expect, borrowing increases welfare by even more when it increases the scale of profitable investment. Due to risk aversion, an entrepreneur may choose to invest less than all his wealth, even when $E(\rho) > r_b$ and the project has only idiosyncratic risk. Sufficient conditions for under-investment in the absence access to risky debt are given in Lemma 2.

Lemma 2: Assume that $u(0) = -\infty$, $W < I_{max}$, and the probability that $\rho = 0$ is positive. Then $\chi < W$.

Proof of Lemma 2: Let p denote the probability that $\rho = 0$. If no money is invested in financial assets, there is a positive probability of zero consumption. This implies expected utility of $-\infty$. Moving a small amount of wealth into the bond market puts a floor on realized consumption, increasing utility. Therefore putting all money into the private investment project cannot be optimal.

The conditions in Lemma 2 are sufficient but not necessary. With enough risk aversion or idiosyncratic risk investment may be scaled back, even without the possibility of zero consumption or the assumption of extreme aversion to low consumption. Following along the lines of Theorem 1, it is straightforward to show that an entrepreneur who is investing less than I_{max} will always borrow fairly priced risky debt, and use part of it to increase the scale of investment.

Appendix B

Proof of Theorem 2: We will demonstrate the existence of, for some parameter values, a pooling equilibrium in which capital structure has an interior optimum. In such a pooling equilibrium, zero profits in the lending market implies that

$$r = \frac{r_b}{(qp_{high} + (1-q)p_{low})}. \tag{B1}$$

The first order condition for the optimal debt level can be rewritten as:

$$\left\{p_j E\left[u'(c_{j,high})\right] + (1-p_j)E\left[u'(c_{j,low})\right]\right\} r_b \tag{B2}$$

$$-p_j E\left[u'(c_{j,high})\right] \frac{r_b}{p_{high}q + p_{low}(1-q)} \geq 0,$$

where $E[u'(c_{j,low})]$ is the conditional expectation of consumption for type j when default occurs, and $E[u'(c_{j,high})]$ is the conditional expectation when debt is repaid in full. As $q \to 1$, the conditions of Theorem 1 hold for both types, and so $\delta = 1$. For q and p_{low} small, the interest rate on debt is no longer fair for the high-type borrower, and in fact can be made arbitrarily large. Clearly then, for combinations of q and p_{low} sufficiently low, at $\delta = 1$ equation (B2) is violated for the high-type borrowers. As δ decreases, the difference between $E[u'(c_{high,low})]$ and $E[u'(c_{high,high})]$ increases. Thus, there exist combinations of δ, q and p_{low} such that equation (B2) holds with equality for the high types. Call δ^* the debt ratio in the pooling equilibrium.

It remains to show that the low types do not deviate from the pooling equilibrium. At the pooled interest rate, they clearly would like to set $\delta = 1$, but are unable to do so without revealing their type. Whether deviations will occur depends on what lenders believe about borrowers who request $\delta \neq \delta^*$. We assume that lenders believe any deviation comes from a low quality entrepreneur. Then a sufficient condition for no deviation by the low types is that their projects have a negative NPV, i.e., $p_{low}\rho < r_b$, because in a separating equilibrium they would not choose to borrow. If $p_{low}\rho \geq r_b$, a necessary condition for a pooling equilibrium is that

$$E[u(c)|\delta = 1, r = \frac{r_b}{p_{low}}] < E[u(c)|\delta = \delta^*, r = \frac{r_b}{p_{high}q + p_{low}(1-q)}]. \tag{B3}$$

Numerical examples verify that equation (B3) holds for some parameter values, but for other parameters, separation occurs. In a separating equilibrium, both types borrow at a fair rate, but the high-type entrepreneurs borrow less.

References

Avery, Robert B., Raphael W. Bostic, and Katharine A. Samolyk, 1998, "The Role of Personal Wealth in Small Business Finance, *Journal of Banking and Finance* 22: 1019-1061

Banerjee, Abhihit and Andrew Newman (1991). "Risk-bearing and the Theory of Income Distribution." *Review of Economic Studies* 58, 211-35.

Berger, Allen N., and Gregory F. Udell (1995). "Relationship Lending and Lines of Credit in Small Firm Finance." *The Journal of Business* 68: 351-81.

___________. (1998), "The Economics of Small Business Finance: The roles of Private Equity and Debt Markets in the Financial Growth Cycle, *Journal of Banking and Finance* 22, 613-673.

Berkowitz, Jeremy and Michele J. White (2002), "Bankruptcy and Small Firms' Access to Credit," NBER Working Paper #9010

Cole, Rebel, J. Wolken and Louise Woodburn, (1996), "Bank and Nonbank Competition for Small Business Credit: Evidence from the 1987 and 1993 National Surveys of Small Business Finances, Federal Reserve Bulletin 82, no 11 983-995.

DeMarzo, Peter; Darrell Duffie (1999), A Liquidity-Based Model of Security Design, *Econometrica*, Vol. 67, No. 1. , pp. 65-99.

Devereux, Michael B. and Gregor W. Smith (1994), "International Risk Sharing and Economic Growth," *International Economic Review*, 35(4), pp. 535-550.

Evans and Jovanovic (1989), "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *The Journal of Political Economy*, Vol. 97, No. 4, pp. 808-827.

Gentry, W. M., and R. G. Hubbard (1998), "Why Do the Wealthy Save So Much: Saving and Investment Decisions of Entrepreneurs," manuscript, Columbia University

Graham, J. R., and C. R. Harvey (1999), "The Theory and Practice of Corporate Finance: Evidence from the Field," manuscript, Duke University. (update reference, look again).

Greenwood, Jeremy and Boyan Jovanovic (1990), "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, 98(5), Part 1, pp. 1076-1107.

Harris, Milton and Artur Raviv (1991), "The Theory of Capital Structure," *Journal of Finance*, 66(1), 297-355.

Hart, Oliver and John Moore, (1994), "A Theory of Debt Based on the Inalienability of Human Capital," *Quarterly Journal of Economics*, Vol. 109, No. 4, pp. 841-879.

Hart, Oliver and John Moore, (1995), "Debt and Seniority: An Analysis of Hard Claims in Constraining Management," *American Economic Review*, 85, 567-585.

Heaton, John and Deborah Lucas (2000), “Portfolio Choice and Asset Prices, The Importance of Entrepreneurial Risk,” *Journal of Finance*.

Holtz-Eakin, Douglas, David Joulfaian and Harvey S. Rosen (1994), “Entrepreneurial Decisions and Liquidity Constraints,” *The RAND Journal of Economics*, Vol. 25, No. 2., pp. 334-347.

Jensen, M.C. and Meckling W. H. (1976), “Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure,” *Journal of Financial Economics*, 3, 305-360.

Kimball, Miles, 1990, “Precautionary Savings in the Small and in the Large,” *Econometrica* 46, 1429-1446.

Leland, Hayne and D. Pyle (1977), “Informational Asymmetries, Financial Structure, and Financial Intermediation,” *Journal of Finance*, Vol 32, No 2, pp. 371-387.

Levy, Amnon, (2000) “Why Does Capital Structure Vary Over the Business Cycle? Theory and Evidence,” manuscript, Northwestern University

Leyer, 1998, “The Present and Future Roles of Banks in Small Business Finance, *Journal of Banking and Finance*, nos 608, 1109-1116

Moskovitz, Toby and Annette Vissing-Jorgensen (2002), “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?” *American Economic Review*

Obstfeld, Maurice (1994), “Risk-Taking, Global Diversification, and Growth*,” American Economic Review*, Vol, 84, Issue 5, 1310-1329

Petersen, Mitchell A. and Raghuran G. Rajan (1994), “The Benefits of Lending Relationships: Evidence from Small Business Data,” *Journal of Finance*, Vol XLIX, No. 1, pp. 3-37

Petersen, Mitchell and Raghuram Rajan (2002), “Does Distance Still Matter? The Information Revolution in Small Business Lending,” *Journal of Finance*, Vol. LVII, No. 6

Poterba, J. M., and L. H. Summers (1995), “A CEO Survey of U.S. Companies’ Time Horizons and Hurdle Rates,” *Sloan Management Review*, Fall, 43-52.

Rampini, Adriano (2004), "Entrepreneurial Activity, Risk, and the Business Cycle." *Journal of Monetary Economics* 51 , 555-573.

Stiglitz, J. and A. Weiss (1981), "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, Vol 71, 3, pp 393-411

Zwiebel, J., (1996), “Dynamic Capital Structure Under Managerial Entrenchment, *American Economic Review*, 86, 1197-1215.

Table 4.1

Hurdle and Fair Debt Rate

Panel A: $\gamma = 2$

Project Risk	p	D/I^*	r debt	Hurdle Rate	C.V. Cons.
Low	0	0.3	0.050	0.120	0.250
Medium	0	0.3	0.050	0.161	0.310
High	0	0.3	0.050	0.250	0.400
Low	0.05	0.3	0.075	0.174	0.217
Medium	0.05	0.3	0.068	0.213	0.285
High	0.05	0.3	0.052	0.298	0.384
Low	0	0.5	0.050	0.120	0.250
Medium	0	0.5	0.050	0.161	0.310
High	0	0.5	0.050	0.250	0.400
Low	0.05	0.5	0.095	0.102	0.189
Medium	0.05	0.5	0.090	0.144	0.272
High	0.05	0.5	0.081	0.234	0.383
Low	0	0.7	0.050	0.120	0.250
Medium	0	0.7	0.050	0.162	0.310
High	0	0.7	0.050	0.250	0.400
Low	0.05	0.7	0.099	0.081	0.166
Medium	0.05	0.7	0.096	0.124	0.259
High	0.05	0.7	0.089	0.216	0.376

Panel B: $\gamma = 0.5$

Project Risk	p	D/I^*	r debt	Hurdle Rate	C.V. Cons.
Low	0	0.3	0.05	0.069	0.262
Medium	0	0.3	0.05	0.081	0.333
High	0	0.3	0.05	0.11	0.451
Low	0.05	0.3	0.093	0.067	0.221
Medium	0.05	0.3	0.091	0.079	0.304
High	0.05	0.3	0.087	0.106	0.434
Low	0	0.5	0.05	0.069	0.262
Medium	0	0.5	0.05	0.081	0.333
High	0	0.5	0.05	0.11	0.451
Low	0.05	0.5	0.099	0.061	0.191
Medium	0.05	0.5	0.098	0.073	0.284
High	0.05	0.5	0.095	0.1	0.421
Low	0	0.7	0.05	0.069	0.262
Medium	0	0.7	0.074	0.078	0.318
High	0	0.7	0.274	0.078	0.318
Low	0.05	0.7	0.101	0.058	0.167
Medium	0.05	0.7	0.1	0.069	0.269
High	0.05	0.7	0.23	0.081	0.336

Table 4.1 continued

Hurdle and Fair Debt Rate

Panel C: $\gamma = 5$

Project Risk	p	D/I^*	r debt	Hurdle Rate	C.V. Cons.
Low	0	0.3	0.050	0.194	0.235
Medium	0	0.3	0.050	0.262	0.285
High	0	0.3	0.050	0.393	0.359
Low	0.05	0.3	0.053	0.526	0.183
Medium	0.05	0.3	0.053	0.533	0.235
High	0.05	0.3	0.053	0.555	0.322
Low	0	0.5	0.050	0.194	0.235
Medium	0	0.5	0.050	0.262	0.285
High	0	0.5	0.050	0.393	0.359
Low	0.05	0.5	0.053	0.526	0.183
Medium	0.05	0.5	0.053	0.533	0.235
High	0.05	0.5	0.053	0.555	0.322
Low	0	0.7	0.050	0.194	0.235
Medium	0	0.7	0.050	0.263	0.285
High	0	0.7	0.050	0.394	0.359
Low	0.05	0.7	0.095	0.133	0.163
Medium	0.05	0.7	0.089	0.217	0.245
High	0.05	0.7	0.078	0.362	0.343

Table 4.2: The Effect of Varying Initial Wealth

$D/I^* = 0.5$, Medium Risk, $p = 0.05$, $I^* = 1$, $\gamma = 2$

W	r debt	Hurdle Rate	C.V. Cons.
1	0.090	0.144	0.272
1.25	0.092	0.123	0.223
1.5	0.094	0.110	0.188
2	0.095	0.095	0.143
5	0.100	0.050	0.058

Table 4.3: Hurdle Rate, Fair Debt Rate, Stock Share

$\gamma = 2$, $W = I^*$

Project Risk	p	D/I^*	r debt	Hurdle	s_0	s	S/W	C.V. Cons	$\rho(c,r_s)$
Low	0	0.3	0.050	0.122	1.371	4.137	1.241	0.287	0.579
Medium	0	0.3	0.050	0.164	1.371	3.943	1.183	0.329	0.465
High	0	0.3	0.050	0.253	1.371	3.620	1.086	0.402	0.328
Low	0.05	0.3	0.072	0.190	1.371	2.430	0.729	0.229	0.413
Medium	0.05	0.3	0.065	0.229	1.371	2.413	0.724	0.288	0.316
High	0.05	0.3	0.053	0.312	1.371	2.383	0.715	0.379	0.223
Low	0	0.5	0.050	0.122	1.371	2.474	1.237	0.287	0.577
Medium	0	0.5	0.050	0.164	1.371	2.360	1.180	0.329	0.464
High	0	0.5	0.050	0.253	1.371	2.181	1.091	0.402	0.330
Low	0.05	0.5	0.094	0.105	1.371	2.361	1.180	0.239	0.671
Medium	0.05	0.5	0.090	0.148	1.371	2.273	1.136	0.297	0.503
High	0.05	0.5	0.080	0.238	1.371	2.133	1.066	0.386	0.339
Low	0	0.7	0.050	0.122	1.371	1.771	1.240	0.287	0.578
Medium	0	0.7	0.050	0.164	1.371	1.689	1.182	0.329	0.465
High	0	0.7	0.050	0.253	1.371	1.560	1.092	0.402	0.330
Low	0.05	0.7	0.099	0.082	1.371	1.860	1.302	0.237	0.758
Medium	0.05	0.7	0.096	0.126	1.371	1.762	1.234	0.293	0.561
High	0.05	0.7	0.089	0.219	1.371	1.616	1.131	0.383	0.368

Table 4.4: Hurdle Rate, Fair Debt Rate, Stock Share

$\gamma = 2$, $W = 1.5 \times I^*$

Project Risk	p	D/I^*	r debt	Hurdle	s_0	s	S/W	C.V. Cons	$\rho(c,r_s)$
Low	0	0.3	0.050	0.100	1.371	2.443	1.303	0.241	0.745
Medium	0	0.3	0.050	0.131	1.371	2.376	1.267	0.266	0.645
High	0	0.3	0.050	0.198	1.371	2.249	1.200	0.315	0.499
Low	0.05	0.3	0.087	0.105	1.371	2.311	1.233	0.220	0.774
Medium	0.05	0.3	0.081	0.137	1.371	2.264	1.207	0.250	0.654
High	0.05	0.3	0.070	0.202	1.371	2.170	1.157	0.307	0.494
Low	0	0.5	0.050	0.100	1.371	1.958	1.305	0.241	0.746
Medium	0	0.5	0.050	0.130	1.371	1.903	1.269	0.266	0.646
High	0	0.5	0.050	0.199	1.371	1.802	1.202	0.315	0.500
Low	0.05	0.5	0.097	0.081	1.371	1.973	1.315	0.218	0.839
Medium	0.05	0.5	0.093	0.112	1.371	1.921	1.280	0.248	0.706
High	0.05	0.5	0.086	0.181	1.371	1.824	1.216	0.304	0.528
Low	0	0.7	0.050	0.100	1.371	1.633	1.306	0.241	0.746
Medium	0	0.7	0.050	0.130	1.371	1.587	1.270	0.267	0.646
High	0	0.7	0.163	0.156	1.371	1.548	1.238	0.288	0.577
Low	0.05	0.7	0.100	0.071	1.371	1.680	1.344	0.215	0.876
Medium	0.05	0.7	0.098	0.102	1.371	1.630	1.304	0.245	0.734
High	0.05	0.7	0.092	0.172	1.371	1.539	1.231	0.300	0.544

Table 6: Hurdle Rate, Fair Debt Rate, Stock Share, Consumption Floor

$\gamma = 2$, $W = 1.5 \times I^*$

Project Risk	p	D/I^*	r debt	Hurdle	s_0	s	S/W	C.V. Cons	$\rho(c,r_s)$
Low	0	0.3	0.050	0.093	1.630	2.947	1.572	0.240	0.803
Medium	0	0.3	0.050	0.119	1.630	2.883	1.538	0.260	0.716
High	0	0.3	0.050	0.178	1.630	2.761	1.472	0.301	0.577
Low	0.05	0.3	0.089	0.095	1.630	2.885	1.539	0.225	0.838
Medium	0.05	0.3	0.084	0.119	1.630	2.831	1.510	0.249	0.736
High	0.05	0.3	0.074	0.177	1.630	2.727	1.455	0.295	0.583
Low	0	0.5	0.050	0.093	1.630	2.361	1.574	0.240	0.803
Medium	0	0.5	0.050	0.119	1.630	2.310	1.540	0.260	0.716
High	0	0.5	0.050	0.178	1.630	2.212	1.475	0.301	0.578
Low	0.05	0.5	0.097	0.075	1.630	2.383	1.589	0.223	0.882
Medium	0.05	0.5	0.094	0.105	1.630	2.337	1.558	0.246	0.772
High	0.05	0.5	0.088	0.163	1.630	2.239	1.493	0.291	0.608
Low	0	0.7	0.050	0.093	1.630	1.971	1.576	0.240	0.804
Medium	0	0.7	0.050	0.119	1.630	1.928	1.542	0.261	0.717
High	0	0.7	0.195	0.133	1.630	1.906	1.525	0.271	0.677
Low	0.05	0.7	0.100	0.068	1.630	2.012	1.610	0.220	0.909
Medium	0.05	0.7	0.098	0.094	1.630	1.967	1.574	0.243	0.794
High	0.05	0.7	0.094	0.145	1.630	1.894	1.515	0.290	0.625

Figure 1

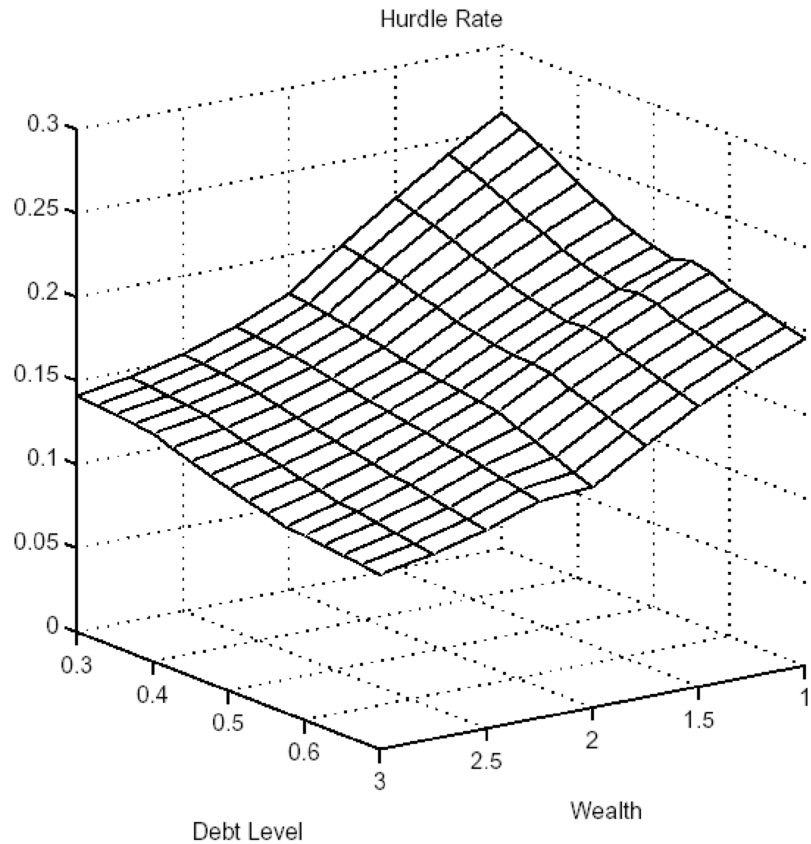
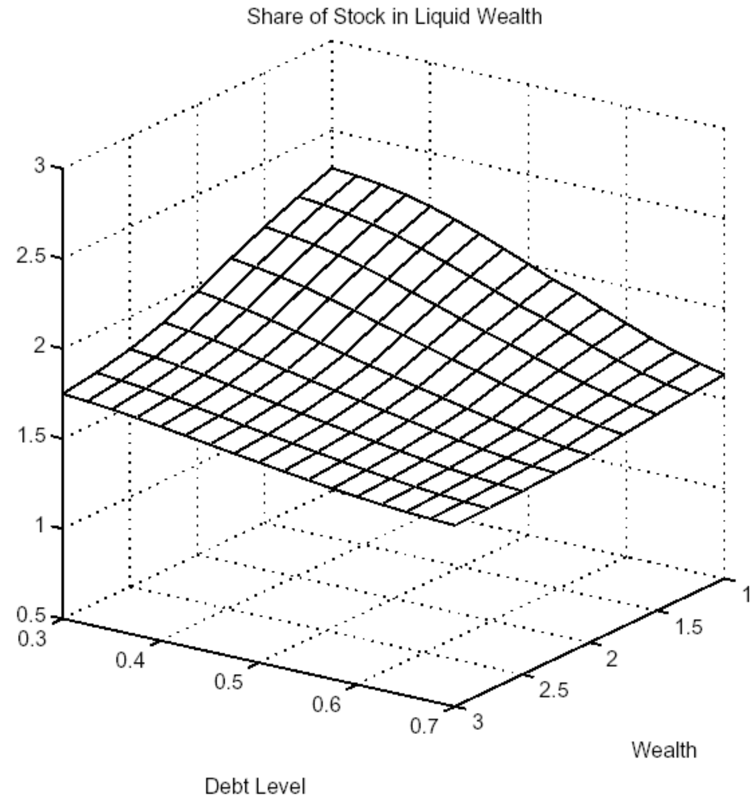

Hurdle Rate
0.3
0.25
0.2
0.15
0.1
0.05
0
0.3
0.4
0.5
0.6
3
2.5
2
1.5
1
Debt Level
Wealth

Figure 2


Share of Stock in Liquid Wealth
3
2.5
2
1.5
1
0.5
0.3
0.4
0.5
0.6
0.7
Debt Level
1
1.5
2
2.5
3
Wealth